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Notes

Bridging the gap: Bargaining with interdependent values

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Abstract

We study dynamic bargaining with asymmetric information and interdependent values. We base our analysis on the equilibria characterized by Deneckere and Liang (2006) for the gap case. We show that as the gap between the cost and value of the weakest type shrinks to zero, the continuous time limit of equilibria changes dramatically from rare bursts of trade with long periods of inactivity to smooth screening over time. In the double limit prices are independent of the shape of the distribution of values. When the uninformed agent's ability to commit to prices disappears so do her rents, yet trade still exhibits delay.

JEL classification: C78; D40

Keywords: Bargaining; Interdependent values; Coase conjecture; Delay

1. Introduction

Deneckere and Liang [4] (henceforth DL)[1] analyze a dynamic bargaining model with interdependent values and one-sided private information. They show that there is generically a unique equilibrium in the gap case (i.e. when there is common knowledge of strict gains from trade).[2] In the non-commitment limit, unless the gap is very large, trade is not immediate. Moreover, this equilibrium has recurring bursts of high probability of agreement, followed by long periods of

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[1] See also the prior work by Evans [5] and Vincent [11].

[2] Interdependent and independent are in reference to the relationship between the value for a given buyer type and the cost of serving those buyer types.

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delay in which the probability of agreement is negligible. Finally, the seller profits are generically positive.

In this paper we consider a sequence of bargaining games with interdependent valuations as the gap (between the lowest value buyer and the cost of serving that buyer) goes to zero. We characterize the properties of the double limit of the sequence of equilibria characterized in DL, taking commitment to zero first and the gap size to zero second. We show that in the limit trade takes place smoothly over time rather than in bursts. One subtlety of the result is that although from the ex-ante perspective for small gaps equilibrium trade is close to smooth (i.e. bursts of trade get arbitrarily small for most types), conditionally on reaching the bottom of the distribution, these bursts of trade remain.

A consequence of smooth trade in the double limit is a major simplification of the equilibrium dynamics in comparison to DL. In equilibrium each type pays a price equal to the cost of serving that type and prices drop slowly over time in a way that is independent of the distribution of types (it depends only on the shape of the cost function and the range of values). That allows closed-form solutions and the possibility of performing comparative static analysis.

We model the case in which the buyer has the private information. An example is car insurance where the buyer knows better its driving ability. A worse driver values insurance more and is more costly to insure. Another example is a farmer selling the rights to shale gas deposits under his land to a specialized energy firm. The firm can estimate better the actual value of the gas in the ground and can exploit it more efficiently than the farmer.[3] For more examples of informed buyers see Burkart and Lee [3]. With a simple transformation, the model can be recast as a durable goods monopolist problem with experience effects so that costs decrease in the volume of cumulative sales (the experience effects make the model mathematically equivalent to the bargaining with interdependent values model). Similarly, the model can be recast as a privately informed seller (as DL do) responding to price offers by an uninformed buyer. For example, a buyer may try to buy a physical asset from a seller and worry about the lemons problem. In this case, to obtain our results, the gap we shrink is the gap between the cost to the seller of the highest quality good (the peach) and the value to the buyer of consuming it.[4]

Looking at the model as a durable good monopolist allows us to compare the limit equilibrium monopolist pricing and the competitive equilibrium, assuming that the competitive firms benefit from industry-wide experience/learning-by-doing effects for example due to lower cost of some common input. It turns out that the limit outcomes under both market structures are the same: both are inefficient, with the same delay of trade and time path of prices. Although price equals marginal cost in both (limit) cases, the competitive equilibrium is inefficient because current firms do not internalize the efficiency gains they accrue to the future producers.[5]

Olsen [9] analyzed the monopolist problem with experience effects directly in the no-gap case. He constructs an equilibrium when demand is linear and marginal costs are linear as a function of cumulative past sales. He also shows the analogy between the competitive equilibrium and monopoly pricing outcomes in his model. We analyze a more general setup and take a limit of equilibria of the gap case. For the case of valuations and costs as in Olsen [9] we show that our limit coincides with the equilibrium he constructed. Although the equilibrium characterized in

3 Again the values are interdependent since the farmer could exploit it himself (albeit more inefficiently) or simply sell it later to another firm whose valuation would likely be correlated with the valuation of the first firm.

4 It is important that strict gains from trade remain with all other types to ensure that all types eventually trade.

5 The detailed analysis is presented in the online Appendix.

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Olsen [9] is qualitatively quite different from the one in DL, they are related because we show that one converges to the other.

Comparisons between the gap and no-gap cases are common in the bargaining literature with independent values. Fudenberg, Levine and Tirole [7] and Gul, Sonnenschein and Wilson [8] have shown that in the gap case there is generically a unique equilibrium.[6] In the non-commitment limit, i.e. as the time for which the uninformed party is able to commit to an offer vanishes, trade is immediate at a price equal to the valuation of the lowest buyer type (informed party). Therefore, in a sequence of games with a decreasing gap and in the corresponding sequence of the non-commitment limits, trade is immediate and seller profits are equal to the gap size, so decrease monotonically to zero. In our case, profits also converge to zero, but not monotonically. The second contrast is that the equilibrium dynamics in our gapless limit are qualitatively different from those for a given gap described in DL.

In the game with no gap, Ausubel and Deneckere [1] show that there are many equilibria. Yet, there exist equilibria that in the non-commitment limit share the characteristics of the gap case: trade is immediate at a price equal to lowest valuation (and hence the seller makes zero profit). This is analogous to our result linking the DL equilibria to the Olsen [9] limit.

2. The model

There is a seller and a buyer. The seller has a unit of an indivisible good. The buyer privately knows his type $c \in [\underline{c} + g, \bar{c}]$, where $g \geqslant 0$ parameterizes the size of the gap. The distribution of possible buyer types c is given by a continuous c.d.f. $F(c; g)$ with density $f(c; g)$ which is continuous and strictly positive for all c and g. When $g > 0$, $F(c; g)$ represents the truncated distribution and $f(c; g) = f(c; 0)/(1 - F(\underline{c} + g; 0))$.[7] F is common knowledge. Additionally, assume that $f(c)$ is Lipschitz continuous, i.e. there exists a Γ such that $\sup_{x,y} |\frac{f(x)-f(y)}{x-y}| \leqslant \Gamma$.

The seller's cost of selling the good to this buyer is c (which the seller does not know). Type c values the good at $v(c)$.[8] The function $v(.)$ is common knowledge, continuously differentiable and satisfies:

$$v(\underline{c}) = \underline{c}, \qquad v'(c) \in [A, B] \quad \text{for all } c$$

for some $A > 1$, $B < \infty$. Note that $A > 1$ implies $v(c) > c$ for all $c > \underline{c}$ and also $v'(c)$ being bounded implies that $\frac{v(c)-\underline{c}}{c-\underline{c}} \in [A, B]$ for all $c > \underline{c}$.

Time is discrete with periods of length Δ and the horizon is infinite. In every period the seller offers a price p, the buyer decides whether to accept current price or reject it. Both players discount payoffs at a rate r, so that if there is trade at time $t \in \{0, \Delta, \ldots\}$ at price p, the payoffs are:

$$\pi_s = (p - c)e^{-rt}, \qquad \pi_b = \big(v(c) - p\big)e^{-rt}$$

As usual, in any (Perfect Bayesian Nash) equilibrium the skimming property holds, that is, if type c accepts with positive probability a price p, then all types $c' > c$ accept that price with probability 1. As a result, for any history of the game the seller's beliefs are a truncated version

6 See also Stokey [10] and Bulow [2].

7 When $g = 0$ we will simplify the notation and let $F(x) = F(x; 0)$.

8 Unlike most of the literature, we call the type of the buyer c, rather than v, but since $v(c)$ is strictly increasing, the costs and values are mapped one-to-one. This yields a simpler notation.

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of the prior $F(c;g)$: there is a cutoff k such that the seller believes that the remaining types are distributed over the range $[\underline{c}+g, k]$ according to $\frac{F(c;g)}{F(k;g)}$.

This cutoff is a natural state variable and is used to define stationary equilibria:

Definition 1. A *stationary equilibrium* is described by a pair of functions $P(k;\Delta)$, $\kappa(p;\Delta)$ which represent the seller's price given the belief cutoff k and buyer acceptance rule (i.e. the cutoff type that accepts price p, which does not depend on the history of the game), such that

(1) Given $\kappa(p;\Delta)$ the seller maximizes his time-zero expected payoffs by following $P(k;\Delta)$
(2) Given the path of prices implied by $P(k;\Delta)$ and $\kappa(p;\Delta)$, for any buyer type c it is optimal to accept prices p if and only if $c \geqslant \kappa(p;\Delta)$

When $g=0$, our model is the same as in Olsen [9] with the exception that he assumes linear F and v.[9] When $g>0$, our model is the same as the one in DL.[10]

3. Gapless limit of DL equilibria

Consider a sequence of games *with a gap* indexed by $g>0$ (with $g\to 0$). Since DL have shown that for any $g>0$, for small Δ their equilibria are close to the limit they characterized, the double limit (i.e. first $\Delta\to 0$ and then $g\to 0$) approximates equilibria for small g as $\Delta\to 0$. The characterization of this double limit is our main result.

3.1. Continuous time limit of DL equilibria

DL provide us with the following result, translated here to our notation[11]:

Proposition 0 *(DL Theorems 2 and 3). For every $g>0$ and Δ the game has a stationary equilibrium. As $\Delta\to 0$, a limit of some stationary equilibria is described by two sequences $\{c_n\}$ and $\{p_n\}$ with $c_0=\underline{c}+g$ and $p_0=v(c_0)$ and the remaining elements defined recursively by*:

$$p_{n+1}=v(c_{n+1})-\frac{(v(c_{n+1})-c_{n+1})^2}{v(c_{n+1})-p_n} \tag{1}$$

$$p_n=E\big[c\,|\,c\in[c_n,c_{n+1}]\big] \tag{2}$$

Given the starting values p_0 and c_0, Eq. (2), which implicitly defines c_{n+1}, is used to compute c_1. Then Eq. (1) is used to compute p_1 and so on. Note that $c_0\leqslant p_0\leqslant c_1\leqslant p_1\ldots.$

The DL limit equilibrium path is derived from these two sequences as follows. Given $\{c_n\}$, let N be the largest n such that $c_n<\bar{c}$. At $t=0$ the seller offers price p_N. It is accepted by types $(c_N,\bar{c}]$. Then there is no trade for some amount of time and the next price is p_{N-1} (the delay is such that type c_N is indifferent between buying immediately at p_N and waiting for the strictly lower price p_{N-1}). Price p_{N-1} is accepted by types $(c_{N-1},c_N]$. This atom of trade is followed

[9] Olsen [9] focuses on a model of a durable goods monopolist with learning by doing effects, but the two problems are mathematically equivalent.

[10] DL actually present a model with the private information on the seller side, but as they point out in Section 7, these models are equivalent.

[11] They also prove that this equilibrium is unique if the distribution of types is discrete.

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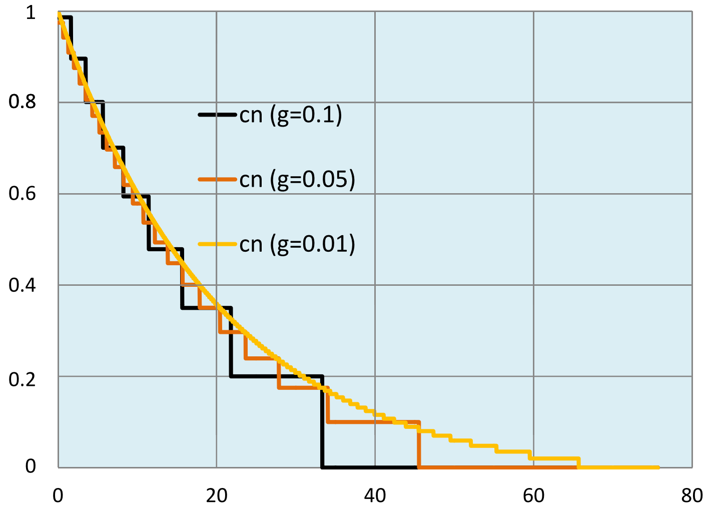

Fig. 1. Cutoffs over time for varying gap sizes.

by another time block of no trade and then price p_{N-2}, accepted by types $(c_{N-2}, c_{N-1}]$, another quiet time block and so on.[12,13]

3.2. *"Bridging the Gap"*

As emphasized in DL, for any $g > 0$, in the limit of $\Delta \to 0$, equilibrium trade is very discontinuous: bursts (atoms) of trade are interrupted by sizable quiet intervals of no trade. In contrast, in Olsen [9], taking the same limit $\Delta \to 0$ results in smooth trade (no atoms, and a positive flow of trade in every instance). We show that as we take $g \to 0$, trade becomes smooth from the ex-ante perspective. (However, conditional on reaching types close to c_0, the equilibrium still resembles the one in DL.) To illustrate below we plot the equilibrium paths for the case in which c is uniform $[0, 1]$ and $v(c) = 2c + g$.[14] We plot the cases $g \in \{0.01, 0.05, 0.1\}$. Fig. 1 (on the top) clearly shows that the paths start looking smoother as g gets smaller but that this is via having more albeit smaller atoms. Fig. 2 shows that conditional on not trading until close to the end, trade will once again look lumpy.

For a given g we now index the DL equilibrium sequences as $\{c_n(g)\}$, $\{p_n(g)\}$.

Proposition 1. *As the gap shrinks, trade becomes smooth*:

$$\lim_{g\to 0} \sup_{n\in S} \left| p_n(g) - c_n(g) \right| = 0 \quad \textit{and} \quad \lim_{g\to 0} \sup_{n\in S} \left| c_{n+1}(g) - c_n(g) \right| = 0$$

where S is a set of indices $S = \{n\colon\ c_n \leqslant \bar{c}\}$ (the set gets larger as $g \to 0$).

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[12] Surprisingly, this limit is non-stationary because the prices in between the bursts of trade change even though the cutoff remains constant. Along the sequence the equilibria are stationary with a positive probability of trade in every period, but that probability over the "quiet periods" goes to zero faster than Δ while slower than Δ around the periods with the burst of activity.

[13] DL show that the equilibrium is generically unique in case the distribution of types is discrete (arbitrarily closely approximating any continuous distribution).

[14] Having $v(c) = 2c + g$ and $c \in [0, 1]$ is equivalent to $v(c) = 2c$ and $c \in [\frac{g}{2}, 1 + \frac{g}{2}]$. This example is slightly different than our model (with both bounds of c changing with g), but the result and reasoning also apply to this case, since the limit behavior depends on the size of the gap at the lowest cost/value.

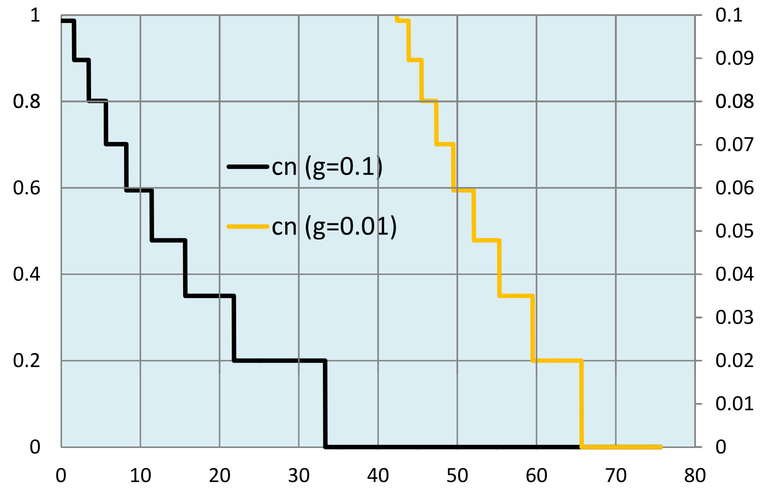

Fig. 2. Conditional atoms remain.

Proof. Recall Eqs. (1) and (2) that describe the DL equilibrium. (1) can be re-written as:

$$p_{n+1} - c_{n+1} = \left(1 - \frac{c_{n+1} - p_n}{v(c_{n+1}) - p_n}\right)(c_{n+1} - p_n)$$

and it implies that:

$$p_{n+1} - c_{n+1} \leqslant c_{n+1} - p_n \tag{3}$$

Eq. (2) can be written as:

$$\int_{c_n}^{c_{n+1}} (p_n - c) f(c)\, dc = 0 \tag{4}$$

Before we consider the general case, for intuition we present a simple case:

(1) Suppose first that $f(c)$ is non-decreasing. Then (4) implies:

$$c_{n+1} \leqslant p_n + (p_n - c_n) = 2p_n - c_n \tag{5}$$

Combining it with (3) yields:

$$c_{n+1} - p_n \leqslant_{\text{by (5)}} p_n - c_n \leqslant_{\text{by (3)}} c_n - p_{n-1} \leqslant_{\text{by (5)}} p_{n-1} - c_{n-1}$$

It implies that the sequence $\{p_n - c_n\}_{n \in S}$ is weakly decreasing. Since $c_0 \to p_0$, the first claim follows. Since $p_n - c_n \to 0$ uniformly for all $n \in S$, by (5) $c_{n+1} \to p_n \to c_n$ (recall $c_{n+1} \geqslant p_n$) uniformly for all $n \in S$ as well, establishing the second claim.

(2) Now consider a general $f(c)$ that satisfies the Lipschitz condition. To provide a simple upper bound on c_{n+1} that solves (4), define:

$$\hat{f}(c) = \begin{cases} \max\{f(\tilde{c}) | \tilde{c} \in [c_n, p_n]\} & \text{if } c \in [c_n, p_n) \\ \min\{f(\tilde{c}) | \tilde{c} \in [p_n, c_{n+1}]\} & \text{if } c \in [p_n, c_{n+1}] \end{cases}$$

Then c_{n+1} is weakly smaller than c'_{n+1} which is the solution to:

$$\int_{c_n}^{p_n} (p_n - c)\hat{f}(c_n)\,dc + \int_{p_n}^{c'_{n+1}} (p_n - c)\hat{f}(c_{n+1})\,dc = 0$$

Using the Lipschitz condition we get:

$$\int_{c_n}^{p_n} (p_n - c)\big(\Gamma(c_{n+1} - c_n) + \hat{f}(c_{n+1})\big)\,dc + \int_{p_n}^{c'_{n+1}} (p_n - c)\hat{f}(c_{n+1})\,dc \geqslant 0$$

which after integrating gives:

$$\big(\Gamma(c_{n+1} - c_n) + \hat{f}(c_{n+1})\big)(p_n - c_n)^2 - \big(c'_{n+1} - p_n\big)^2 \hat{f}(c_{n+1}) \geqslant 0$$

Which can be written as

$$K(p_n - c_n)^2 + (p_n - c_n)\frac{(p_n - c_n)}{(c_{n+1} - c_n)} - \big(c'_{n+1} - p_n\big)\frac{(c'_{n+1} - p_n)}{(c_{n+1} - c_n)} \geqslant 0$$

for $K \equiv \frac{\Gamma}{\min_c f(c)}$.

Since $\frac{(p_n - c_n)}{(c_{n+1} - c_n)} < 1$ and $c_n \leqslant p_n \leqslant c_{n+1} \leqslant c'_{n+1}$ we obtain:

$$c_{n+1} - p_n \leqslant p_n - c_n + K(p_n - c_n)^2 \tag{6}$$

Combining it with (3) yields:

$$c_{n+1} - p_n \leqslant p_n - c_n + K(p_n - c_n)^2 \leqslant_{\text{by (3)}} c_n - p_{n-1} + K(c_n - p_{n-1})^2$$

Define a sequence $\{y_n\}_{n=1}^{M(g)}$ as follows:

$$y_n = c_n - p_{n-1}$$
$$M(g) = \min\left\{N\colon \sum_{n=1}^{N} y_n \geqslant \bar{c} - (\underline{c} + g)\right\}$$

$M(g)$ keeps track of how many "atoms" are needed to cover the whole domain $[\underline{c} + g, \bar{c}]$: note that the range $[\underline{c} + g, \bar{c}]$ is covered by non-overlapping segments $[c_k, p_k] \cup [p_k, c_{k+1}]$ and the length of the $[p_k, c_{k+1}]$ segments is y_{k+1}. Therefore there are at most $M(g) + 1$ elements in S.[15] To establish the claim it is sufficient to prove that all the elements $\{y_n\}_{n=1}^{M(g)}$ converge to 0 as $g \to 0$.

Instead of doing it directly, we define a sequence $\{x_n\}_{n=1}^{N(g)}$ as follows:

$$x_1 = c_1 - p_0$$
$$x_{n+1} = x_n + K x_n^2$$
$$N(g) = \min\left\{N\colon \sum_{n=1}^{N} x_n \geqslant \bar{c} - (\underline{c} + g)\right\}$$

15 Actually there are fewer elements since in the definition of $M(g)$ we do not count the segments $[c_k, p_k]$.

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In general, by inequality (6) $y_n \leqslant x_n$ (and if (6) holds exactly, the two sequences are identical). In Appendix A in a technical Lemma 1 we show that for every g[16]:

$$\max_{n \leqslant M(g)} y_n \leqslant x_{N(g)+1}$$

Since x_n is an increasing sequence, this observation allows us to establish the claims by proving that:

$$\lim_{g \to 0} x_{N(g)+1} = 0$$

because that yields a uniform convergence of all y_n to zero as $g \to 0$.

Note that $\lim_{g \to 0} x_n = 0$ for any finite n, so for the limit to be positive, we need $N(g) \to \infty$.

Suppose there exists $\varepsilon \in (0, 1)$ such that for arbitrarily small g, $x_{N(g)} \geqslant \varepsilon$. Since x_{n+1} is a quadratic function of x_n, we have

$$x_n = \frac{x_{n+1}}{1 + K x_n} \geqslant \frac{x_{n+1}}{1 + K x_{n+1}}$$

Therefore, if $x_{N(g)} \geqslant \varepsilon$ then

$$x_{N(g)-1} \geqslant \frac{\varepsilon}{1 + K\varepsilon} \geqslant \frac{\varepsilon}{1 + K}$$

In general:

$$x_{N(g)-n} \geqslant \frac{\frac{\varepsilon}{1+(n-1)K}}{1 + K\frac{\varepsilon}{1+(n-1)K}} = \frac{\varepsilon}{1 + (n-1)K + K\varepsilon} \geqslant \frac{\varepsilon}{1 + nK}$$

In words, if $x_{N(g)}$ is large, then for small n, all $x_{N(g)-n}$ are proportionally large too. It turns out that the sum of these lower bounds is diverging:

$$\sum_{n=0}^{\infty} \frac{1}{1 + Kn} = \infty$$

Therefore, having $x_{N(g)} \geqslant \varepsilon$ would imply that $N(g)$ is bounded even for arbitrarily small g. But that is a contradiction since we have argued already that x_n converges to zero for all finite n. □

Does this double limit correspond to equilibria of the no-gap case, $g = 0$, as $\Delta \to 0$? In general we do not know, but it does coincide with the limit of the equilibrium constructed in Olsen [9] in case of linear F and v. We conjecture that it is the case for other F and v as well. Our result does not imply it because DL show point-wise convergence of equilibria for every $g > 0$ and for our proof we would need uniform convergence (in other words, we did not show that the order of taking the limits $g \to 0$ and $\Delta \to 0$ does not matter).[17]

[16] This claim requires a proof because even though $y_n \leqslant x_n \leqslant x_{N(g)}$, for any $n \leqslant N(g)$, there are potentially more elements in the y sequence: $M(g) \geqslant N(g)$.

[17] Establishing such a result is beyond the scope of this paper because it would require a non-trivial extension of the proof in DL. Their proof uses backward induction from the time bargaining ends. When $g > 0$, in any equilibrium bargaining ends in finite time. However, as $g \to 0$ that time grows without bound and hence regular induction arguments are not sufficient to establish uniform convergence.

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3.3. Dynamics of the atomless limit

The double limit is an approximation of equilibria for small g as $\Delta \to 0$. We can use Proposition 1 to fully characterize that approximation:

1. (**No profits in the limit**) Eq. (2) implies that in the DL equilibrium, types accepting price p_n for $n < N$ cost the seller on average p_n. Therefore, the seller makes on average no profit beyond the first offer p_N at $t = 0$ (which yields a positive expected profit since generically $c_{N+1} > \bar{c}$). A corollary of our result that atoms disappear as the gap shrinks is that (in the double limit) the seller's profit converges to zero.
2. (**Price equal to cost**) The first part of the proposition means that every type pays a price equal to c, the cost of serving that type. Therefore, in the (double) limit, the strategy of the seller converges to:

$$P(k) = k$$

In the gap case (and in general, if either g or Δ are away from zero), equilibrium prices are quite complicated and depend on the details of the distribution. As the gap and commitment disappear, prices become very simple and independent of the distribution.
3. (**Time to trade**) We can also pin down $K(t)$, which is defined as the time-path of the equilibrium cutoff in the limit. In the DL equilibrium each type is indifferent between trading now and trading one period later. Since we know that the prices that each type pays converge to $P(t) = K(t)$ we can use "revelation principle" arguments: an agent that is supposed to trade at $K(t)$ can pretend to be a different type, that trades at time $\hat{t}$. The first-order condition of "truth telling" is[18]:

$$\frac{\partial}{\partial \hat{t}}\left(e^{-r\hat{t}}\left[v\left(K(t)\right) - K(\hat{t})\right]\right)\Big|_{\hat{t}=t} = 0$$

This condition implies that $K(t)$ satisfies a differential equation:

$$r\left(v\left(K(t)\right) - K(t)\right) = -\dot{K}(t)$$

where $\frac{\partial p_t}{\partial t} = \dot{K}(t)$ is the speed at which prices drop over time. The boundary condition is $K(0) = \bar{c}$ (since there is no atom at time 0), and that uniquely defines $K(t)$. It is also clear from the expressions above that the relevant notion of time in model is rt so if r is twice as high it takes half the amount of real time to reach the same cutoff.
4. (**Distribution independent**) Since the differential equation defining $K(t)$ and prices $P(k)$ are independent of the distribution of types, the equilibrium time-path of prices is also independent of the distribution.

One can exploit these properties and the relative simple nature of the equilibrium in the limit to establish comparative static results.[19] For example, since the outcome is independent of the distribution one can easily show:

[18] Since $v(c)$ is increasing and $K(t)$ is decreasing, the problem in t and $\hat{t}$ is supermodular. Hence, the first-order condition identifies a global optimum (since all times are reached on the equilibrium path, there are no other deviations for the buyer).

[19] See Fuchs and Skrzypacz [6] for other comparative static results in a related model.

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Proposition 2. *Given two distributions of buyer's values F and H such that F first order stochastically dominates H, compare the double limit of equilibria as $\Delta \to 0$ and $g \to 0$. The expected time to trade is longer and average prices are lower if the distribution of values is H.*

If we assume $v(c) = \eta c$ for some $\eta > 1$, let $\underline{c} \to 0$ (so that $v(\underline{c}) \to \underline{c}$) and we normalize $\bar{c} = 1$ then we get:

$$K(t) = e^{-r(\eta-1)t} = p_t$$

That implies comparative statics:

Proposition 3. *Suppose $v(c) = \eta c$ with $\eta > 1$ and consider the limit $\Delta \to 0$ and $\underline{c} \to 0$ (so that $v(\underline{c}) \to \underline{c}$). Then for any $t > 0$, $K(t)$ and p_t decrease in η and in r.*

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Appendix A

Lemma 1. *Fix $x_1 > 0$ and let $x_{n+1} = x_n + Kx_n^2$ for some $K > 0$. Let $y_1 = x_1$ and y_n be a sequence that satisfies $y_n \geqslant 0$ and $y_{n+1} \leqslant y_n + Ky_n^2$.*

Let $N(g) = \min\{N\colon \sum_{n=1}^{N} x_n \geqslant \bar{c} - (\underline{c} + g)\}$ and $M(g) = \min\{N\colon \sum_{n=1}^{N} y_n \geqslant \bar{c} - (\underline{c} + g)\}$.[20] *Then $\max_{n \leqslant M(g)} y_n \leqslant x_{N(g)+1}$.*

Proof. For the sake of contradiction, suppose that $\max_{n \leqslant M(g)} y_n > x_{N(g)+1}$. Let $n_0 = \min\{n \leqslant M(g)\colon\ y_n > x_{N(g)+1}\}$. It must be the case that $y_{n_0-1} > x_{N(g)}$ since otherwise $y_{n+1} \leqslant y_n + Ky_n^2$ would imply $y_{n_0} \leqslant x_{N(g)+1}$. Moreover, it must be the case that $n_0 - 1 > N(g)$ because $y_n \leqslant x_n \leqslant x_{N(g)}$ for all $n \leqslant N(g)$.

Inductively, we must have $y_{n_0-k} > x_{N(g)-k+1}$ for all $k \leqslant N(g)$.

Then:

$$\sum_{n=1}^{n_0-1} y_n \geqslant \sum_{n=n_0-N(g)}^{n_0-1} y_n > \sum_{n=1}^{N(g)} x_n \geqslant \bar{c} - (\underline{c} + g)$$

That leads to a contradiction of the definition of $M(g)$ (since already the sum of the first $n_0 - 1$ of the y_n sequence satisfies the condition, it is not true that $n_0 \leqslant M(g)$). □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jet.2013.01.002.

[20] If the minimum does not exist then $M(g) = \infty$.

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